Harmonic-Killing vector fields on Kähler manifolds

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The Lie algebra action of $\Gamma(TM)$ on connections on M, $(Con(M))$, is given by

$$Con(M)\times\Gamma(TM) \to Con(M)\,, \quad (\nabla, X) \mapsto \mathcal{L}_X\nabla\,.$$

Namely,

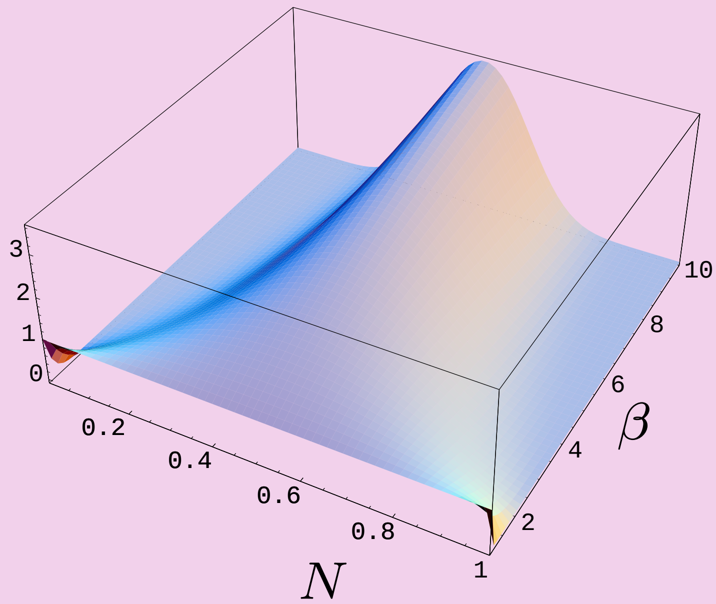
$$\mathcal{L}_X\nabla = ev|_{t=0} \circ \frac{\partial}{\partial t} \circ \nabla^{\varphi_t},$$

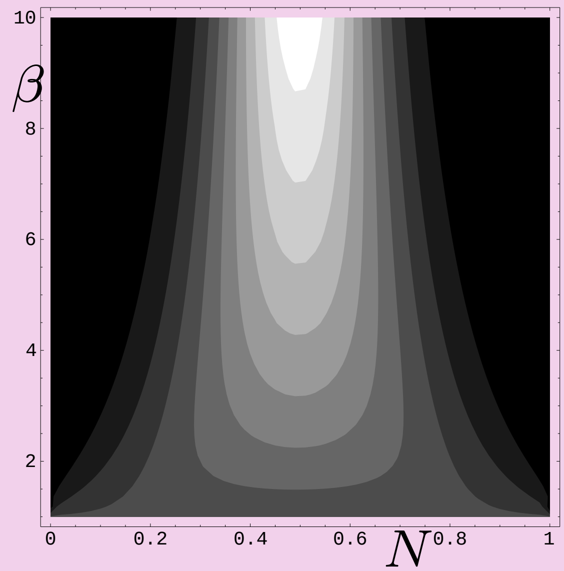
where ∇^{φ_t} is the result of the natural action of $\Gamma(TM)$ on $Con(M)$, that is,

$$\nabla_Z^{\varphi_t} Y = \varphi_t^* \circ (\nabla_{Z^{\varphi_{-t}}} Y^{\varphi_{-t}}) \circ \varphi_{-t}^*,$$

and by W^φ we denote the right action of Diff(M) on $\Gamma(TM)$, i.e., $W^\varphi = \varphi^* \circ W \circ \varphi^{-1*}$, $\varphi \in$Diff(M), $W \in \Gamma(TM)$.

For more details see http://www.ma.umist.ac.uk/kd/PREPRINTS/HKaehler.pdf





For more details see artex.pdf

Theorem

M complete, connected, simply-connected

$\exists$ naturally reductive homogeneous structure

$\Leftrightarrow T_{XYZ} + T_{YXZ} = 0 \qquad \forall X, Y, Z \in \mathcal{M}$

Theorem

$T = D - \nabla$ is naturally reductive

$\Leftrightarrow \nabla R = 0.$